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A Hierarchical Hypergraph and Superhypergraph Framework for Semantic and Behavioral Graphs in Psychology and the Social Sciences

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Abstract

Graph theory—modeling entities as vertices and their relationships as edges—has been applied across domains from anatomical networks (e.g. teeth) to social systems [1, 2]. In psychology and the social sciences, Behavior Graphs capture temporal sequences of actions or states, while Semantic Graphs represent conceptual associations underlying memory and cognition. Here, we extend both models using HyperGraphs and SuperHyperGraphs to create hierarchical, multi-scale representations. This framework enables nested modeling of cognitive and behavioral structures, offering a versatile approach for analyzing complex phenomena in psychological and social research.

Keywords: Superhypergraph, Hypergraph, Semantic Graph, Behavior graph.

1|Introduction

We fix notation and recall key definitions used throughout this paper. Unless otherwise stated, all graphs are finite. For comprehensive treatments, see the cited references.

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1.1| Hypergraphs and SuperHyperGraphs

Graph theory models anatomical structures such as teeth and their spatial or functional relationships by representing each tooth as a vertex and each connection as an edge [1, 2]. A *hypergraph* extends this notion by allowing each *hyperedge* to connect any nonempty subset of vertices simultaneously [3, 4, 5]. HyperGraphs have been applied across various fields, including artificial intelligence and chemistry [6, 7, 8, 9, 10]. Building on this, a *SuperHyperGraph* employs iterative powerset constructions to build hierarchical layers of vertices and hyperedges, thereby capturing nested, multi-scale relationships [11, 12, 13, 14, 15]. Here, the integer $n \geq 0$ denotes the depth of powerset iteration.

Definition 1.1 (Graph)**.** [16] A *graph* is an ordered pair $G = (V, E)$ where

- V is a finite set of *vertices*,
- $E \subseteq V \times V$ is a set of unordered or ordered pairs of distinct vertices, called *edges*.

Example 1.2 (Social Network Graph)**.** (cf.[17]) Let $V = \{\text{Ayano}, \text{Jose}, \text{Taro}, \text{Ziro}\}$ represent four individuals, and

$$E = \big\{\{\text{Ayano}, \text{Jose}\},\ \{\text{Jose}, \text{Taro}\},\ \{\text{Taro}, \text{Ziro}\}\big\}$$

represent mutual "friendship" ties. Then

$$G = (V, E)$$

is a graph modeling the social connections among these four people.

Definition 1.3 (Base Set)**.** Let V_0 be a finite set, called the *base set*. All subsequent constructions are drawn from V_0 or its iterated powersets.

Definition 1.4 (Powerset)**.** [18] For any set X, its *powerset* is

$$\mathcal{P}(X) \ = \ \big\{\, A \mid A \subseteq X \big\}.$$

Definition 1.5 (Hypergraph)**.** [3, 4] A *hypergraph* is a pair $H = (V, E)$ where

- V is a finite set of *vertices*,
- $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is a finite family of nonempty subsets of V, called *hyperedges*.

Example 1.6 (Major Depressive Symptom Hypergraph)**.** (cf.[19, 20]) Let V be the set of common symptoms of major depression:

$$V = \{\,\text{Sadness},\ \text{Anhedonia},\ \text{Fatigue},\ \text{Sleep Disturbance},$$
$$\text{Appetite Change},\ \text{Weight Loss},\ \text{Concentration Impairment}\}.$$

Define the hyperedge set

$$E = \big\{\ \{\text{Sadness, Anhedonia, Fatigue}\},$$
$$\{\text{Sleep Disturbance, Fatigue, Concentration Impairment}\},\ \{\text{Appetite Change, Weight Loss}\}\big\}.$$

Then

$$H = (V, E)$$

is a hypergraph modeling symptom clusters in major depression. Each hyperedge groups together symptoms that frequently co-occur, reflecting underlying latent dimensions such as mood, neurovegetative, and cognitive domains.

Definition 1.7 (n-th Powerset)**.** [21, 22, 23] Define inductively for $k \geq 0$:

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big).$$

We write $\mathcal{P}_n(V_0)$ for $\mathcal{P}^n(V_0)$ and denote by $\mathcal{P}_n^*(V_0)$ its collection of nonempty subsets.

Example 1.8 (Second Powerset of Basic Emotions)**.** Let the base set of primary emotions be

$$V_0 = \{\,\text{Joy},\ \text{Sadness},\ \text{Fear},\ \text{Anger}\}.$$

Then the first powerset is

$$\mathcal{P}^1(V_0) = \mathcal{P}(V_0) = \{\,\{\text{Joy}\},\ \{\text{Sadness}\},\ \{\text{Fear}\},\ \{\text{Anger}\},\ \{\text{Joy}, \text{Sadness}\},\ \ldots,\ \{\text{Joy}, \text{Sadness}, \text{Fear}, \text{Anger}\}\}.$$

The second powerset is

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big),$$

whose elements are all nonempty collections of subsets of V_0. For instance:

$$\big\{\{\text{Joy}\}, \{\text{Sadness}\}\big\} \quad \text{and} \quad \big\{\{\text{Fear}, \text{Anger}\}, \{\text{Joy}, \text{Fear}\}\big\}$$

are two representative members of $\mathcal{P}^2(V_0)$. Such higher-order collections can model cognitive groupings of emotion-clusters in hierarchical analyses.

Notation 1.9 (Iterated powerset)**.** *For a set X and $k \in \mathbb{N}_0$, define*

$$\mathcal{P}^{\,0}(X) := X, \qquad \mathcal{P}^{\,k+1}(X) := \mathcal{P}\big(\mathcal{P}^{\,k}(X)\big).$$

We call the elements of $\mathcal{P}^{\,n}(X)$ the level-n *objects over X.*

Definition 1.10 (n-SuperHyperGraph)**.** [24, 25, 26, 27] Let V_0 be a nonempty base set and $n \in \mathbb{N}$. An *n-SuperHyperGraph* is a pair

$$\text{SuHyG}^{(n)} = (V, E),$$

where the *n-supervertex set* V satisfies $V \subseteq \mathcal{P}^{\,n}(V_0)$, and the *$n$-superedge set* is a family of nonempty subsets of V:

$$E \ \subseteq\ \mathcal{P}(V) \setminus \{\emptyset\}.$$

Thus, each n-superedge $e \in E$ is a set whose elements are n-supervertices (i.e. elements of $V \subseteq \mathcal{P}^{\,n}(V_0)$).

Remark 1.11 (Why $E \subseteq \mathcal{P}(V)$ is the correct typing)**.** In a (super)hypergraph, edges are *collections of vertices*. Since $V \subseteq \mathcal{P}^{\,n}(V_0)$ already fixes the type of n-supervertices, edges must live one powerset level above V, namely inside $\mathcal{P}(V)$. Writing $E \subseteq \mathcal{P}^{\,n}(V_0)$ would incorrectly type edges as *vertices* rather than sets of vertices.

Example 1.12 (Depressive Symptom 2-SuperHyperGraph)**.** Let the base set of common major-depressive symptoms be

$$V_0 = \{\text{Sadness, Anhedonia, Fatigue, Sleep Disturbance, Appetite Change, Concentration Impairment}\}.$$

Define level-1 hyperedges (symptom clusters) by

$$E^{(1)} = \Big\{e_{\text{mood}} = \{\text{Sadness}, \text{Anhedonia}, \text{Fatigue}\},$$
$$e_{\text{sleep}} = \{\text{Sleep Disturbance}, \text{Fatigue}, \text{Concentration Impairment}\},$$
$$e_{\text{somatic}} = \{\text{Appetite Change}, \text{Fatigue}\}\Big\}.$$

Set the level-2 supervertex set to be the full level-2 objects,

$$V_2 := \mathcal{P}^{\,2}(V_0) = \mathcal{P}\big(\mathcal{P}(V_0)\big),$$

and define the level-2 superedges by "lifting" each level-1 edge to all of its nonempty subcollections:

$$E_2 := \{\,\mathcal{P}(e) \setminus \{\emptyset\}\ \mid\ e \in E^{(1)}\,\}\ \subseteq\ \mathcal{P}(V_2).$$

Two representative level-2 superedges are

$$\mathcal{P}(e_{\text{mood}}) \setminus \{\emptyset\} = \big\{\{\text{Sadness}\}, \{\text{Anhedonia}\}, \ldots, \{\text{Sadness}, \text{Anhedonia}, \text{Fatigue}\}\big\},$$
$$\mathcal{P}(e_{\text{sleep}}) \setminus \{\emptyset\} = \big\{\{\text{Sleep Disturbance}\}, \{\text{Fatigue}\}, \ldots,$$
$$\{\text{Sleep Disturbance}, \text{Fatigue}, \text{Concentration Impairment}\}\big\}.$$

Each displayed superedge is a subset of V_2; hence $E_2 \subseteq \mathcal{P}(V_2)$ as required.

Example 1.13 (Cognitive Function 3-SuperHyperGraph)**.** Let the base set of core cognitive domains be

$$V_0 = \{\text{Memory}, \ \text{Attention}, \ \text{ExecutiveFunction}\}.$$

Define two level-1 hyperedges

$$e_1 = \{\text{Memory}, \text{Attention}\}, \qquad e_2 = \{\text{Attention}, \text{ExecutiveFunction}\}.$$

Form their level-2 lifts

$$\mathcal{P}(e_1) \setminus \{\emptyset\} = \big\{\{\text{Memory}\}, \{\text{Attention}\}, \{\text{Memory}, \text{Attention}\}\big\},$$

and analogously for e_2. For $n = 3$, take

$$V_3 := \mathcal{P}^3(V_0), \qquad E_3 := \left\{ \ \mathcal{P}^2(e) \setminus \{\emptyset\} \ \middle| \ e \in \{e_1, e_2\} \ \right\} \ \subseteq \ \mathcal{P}(V_3),$$

where $\mathcal{P}^2(e) = \mathcal{P}\big(\mathcal{P}(e)\big)$. Two representative level-3 superedges are

$$\mathcal{P}^2(e_1) \setminus \{\emptyset\} = \big\{\{\{\text{Memory}\}, \{\text{Attention}\}\}, \ \{\{\text{Memory}, \text{Attention}\}\}, \ \ldots\big\},$$

$$\mathcal{P}^2(e_2) \setminus \{\emptyset\} = \big\{\{\{\text{Attention}\}, \{\text{ExecutiveFunction}\}\}, \ \{\{\text{Attention}, \text{ExecutiveFunction}\}\}, \ \ldots\big\}.$$

Thus $E_3 \subseteq \mathcal{P}(V_3)$, i.e. every level-3 superedge is a set of level-3 supervertices.

2| Behavior Graph and Their Extensions

A Behavior Graph represents observed actions or states as nodes, with edges indicating transitions or sequences between behaviors over time (cf.[28, 29, 30, 31, 32]). In this section, we explore their extensions using HyperGraphs and SuperHyperGraphs.

Definition 2.1 (Behavior Graph)**.** A *Behavior Graph* in the social sciences is a directed, weighted graph

$$G = (V, E, f, p),$$

where

- V is a finite set of *behavioral states* (e.g. actions or observed events),
- $E \subseteq V \times V$ is the set of *transitions* between states,
- $f : E \to \mathbb{N}$ assigns each transition $(u, v) \in E$ a *frequency* of occurrence,
- $p : E \to [0, 1]$ assigns each transition $(u, v) \in E$ a *probability* or normalized weight, with

$$p(u, v) \ = \ \frac{f(u, v)}{\sum_{w\,:\,(u,w)\in E} f(u, w)}.$$

Example 2.2 (Smartphone Usage Behavior Graph)**.** Let

$$V = \{\,\text{UnlockPhone}, \ \text{ViewNotifications}, \ \text{OpenApp}, \\ \text{SendMessage}, \ \text{LockPhone}\},$$

and

$$E = \big\{\,(\text{UnlockPhone}, \text{ViewNotifications}), \ (\text{UnlockPhone}, \text{OpenApp}), \\ (\text{ViewNotifications}, \text{OpenApp}), \ (\text{ViewNotifications}, \text{LockPhone}), \\ (\text{OpenApp}, \text{SendMessage}), \ (\text{OpenApp}, \text{LockPhone}), \ (\text{SendMessage}, \text{LockPhone})\big\}.$$

Define the frequency function $f \colon E \to \mathbb{N}$ by

$$\begin{aligned} f(\text{UnlockPhone}, \text{ViewNotifications}) &= 120, & f(\text{UnlockPhone}, \text{OpenApp}) &= 80, \\ f(\text{ViewNotifications}, \text{OpenApp}) &= 100, & f(\text{ViewNotifications}, \text{LockPhone}) &= 20, \\ f(\text{OpenApp}, \text{SendMessage}) &= 50, & f(\text{OpenApp}, \text{LockPhone}) &= 30, \\ f(\text{SendMessage}, \text{LockPhone}) &= 50. \end{aligned}$$

The transition probability function $p\colon E \to [0,1]$ is then

$$p(u,v) \;=\; \frac{f(u,v)}{\sum_{w\,:\,(u,w)\in E} f(u,w)}.$$

For example,

$$p(\text{UnlockPhone}, \text{ViewNotifications}) = \frac{120}{120+80} = 0.60,$$
$$p(\text{OpenApp}, \text{SendMessage}) = \frac{50}{50+30} = 0.625.$$

Hence

$$G = (V, E, f, p)$$

is a Behavior Graph modeling typical transitions users perform during a smartphone session.

Example 2.3 (E-Commerce Session Behavior Graph)**.** An E-Commerce Session is a continuous sequence of user interactions with an online store, typically ending with inactivity or checkout (cf.[33, 34]). Let

$$V = \{\, \text{ViewProduct},\ \text{AddToCart},$$
$$\text{RemoveFromCart},\ \text{Checkout},\ \text{Exit}\},$$

and

$$E = \Big\{\, (\text{ViewProduct}, \text{AddToCart}),\ (\text{ViewProduct}, \text{Exit}),\ (\text{AddToCart}, \text{Checkout}),\ (\text{AddToCart}, \text{RemoveFromCart}),$$
$$(\text{RemoveFromCart}, \text{AddToCart}),\ (\text{RemoveFromCart}, \text{Exit}),\ (\text{Checkout}, \text{Exit})\Big\}.$$

Define the frequency function $f\colon E \to \mathbb{N}$ by

$$\begin{aligned} f(\text{ViewProduct}, \text{AddToCart}) &= 40, & f(\text{ViewProduct}, \text{Exit}) &= 160,\\ f(\text{AddToCart}, \text{Checkout}) &= 25, & f(\text{AddToCart}, \text{RemoveFromCart}) &= 15,\\ f(\text{RemoveFromCart}, \text{AddToCart}) &= 5, & f(\text{RemoveFromCart}, \text{Exit}) &= 15,\\ f(\text{Checkout}, \text{Exit}) &= 25. \end{aligned}$$

The transition probability function $p\colon E \to [0,1]$ is then

$$p(u,v) \;=\; \frac{f(u,v)}{\sum_{w\,:\,(u,w)\in E} f(u,w)}.$$

For instance,

$$p(\text{ViewProduct}, \text{AddToCart}) = \frac{40}{40+160} = 0.20, \quad p(\text{AddToCart}, \text{Checkout}) = \frac{25}{25+15} = 0.625.$$

Hence

$$G = (V, E, f, p)$$

is a Behavior Graph modeling typical user transitions during an online shopping session.

A Behavior HyperGraph models behavioral states as vertices, hyperedges grouping co-occurring states or joint transitions, with weights aggregating transition frequencies. A Behavior SuperHyperGraph hierarchically lifts a Behavior HyperGraph via iterated powersets, forming level-n vertices and superedges from hyperedge subcollections systematically. We state these definitions below.

Definition 2.4 (Behavior HyperGraph)**.** Let $G = (V, E_G, f, p)$ be a Behavior Graph. A *Behavior HyperGraph* is a tuple

$$H = \big(V,\, E_H,\, F,\, P\big),$$

where

- V is the same finite set of behavioral states as in G,
- $E_H \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is a family of nonempty subsets of V, called *hyperedges*, each representing a frequently co-occurring set of states,

- $F : E_H \to \mathbb{N}$ assigns each hyperedge e its *aggregate frequency* $F(e) = \displaystyle\sum_{(u,v)\in e\times e,\ u\neq v} f(u,v)$,
- $P : E_H \to [0,1]$ assigns each e a *normalized weight* $P(e) = \dfrac{F(e)}{\max_{e'\in E_H} F(e')}$.

Example 2.5 (Workplace Communication Behavior HyperGraph)**.** Consider the set of behavioral states for workplace communication:

$$V = \{\,\text{Email},\ \text{Chat},\ \text{Meeting},\ \text{DocumentShare}\}.$$

Suppose the Behavior Graph $G = (V, E_G, f, p)$ records the following transition frequencies $f(u,v)$:

$f(u,v)$	Email	Chat	Meeting	DocumentShare
Email	˘	30	15	0
Chat	20	˘	12	25
Meeting	10	7	˘	5
DocumentShare	0	20	8	˘

All other $f(u,v)$ not listed are zero.

The induced Behavior HyperGraph $H = (V, E_H, F, P)$ uses:

$$E_H = \big\{\{u,v\} : (u,v)\in E_G\big\}\ \cup\ \big\{\{c\}\cup N_c^{\text{out}} : |N_c^{\text{out}}| \geq 2\big\},$$

where $N_c^{\text{out}} = \{\,v : (c,v)\in E_G\}$. Concretely,

$$\begin{aligned}E_H = \{\,&\{\text{Email}, \text{Chat}\},\ \{\text{Email}, \text{Meeting}\},\ \{\text{Chat}, \text{Meeting}\},\ \{\text{Chat}, \text{DocumentShare}\},\\ &\{\text{Meeting}, \text{DocumentShare}\},\\ &\{\text{Email}, \text{Chat}, \text{Meeting}\},\ \{\text{Chat}, \text{Email}, \text{DocumentShare}\},\\ &\{\text{Meeting}, \text{Email}, \text{DocumentShare}\},\ \{\text{DocumentShare}, \text{Chat}, \text{Meeting}\}\}.\end{aligned}$$

The aggregate frequency F on each hyperedge e is

$$F(e) = \sum_{\substack{(u,v)\in E_G\\ u,v\in e}} f(u,v),$$

and the normalized weight is

$$P(e) = \frac{F(e)}{\max_{e'\in E_H} F(e')}.$$

A few values are:

$$F\big(\{\text{Email}, \text{Chat}\}\big) = 30 + 20 = 50 \qquad P\big(\{\text{Email}, \text{Chat}\}\big) = \tfrac{50}{95} \approx 0.526$$

$$F\big(\{\text{Chat}, \text{DocumentShare}\}\big) = 25 + 20 = 45 \qquad P\big(\{\text{Chat}, \text{DocumentShare}\}\big) = \tfrac{45}{95} \approx 0.474$$

$$F\big(\{\text{Email}, \text{Chat}, \text{Meeting}\}\big) = 30+20+15+10+12+7 = 94 \qquad P\big(\{\text{Email}, \text{Chat}, \text{Meeting}\}\big) = \tfrac{94}{95} \approx 0.989$$

$$F\big(\{\text{Chat}, \text{Email}, \text{DocumentShare}\}\big) = 20 + 30 + 25 + 20 = 95 \qquad P\big(\{\text{Chat}, \text{Email}, \text{DocumentShare}\}\big) = 1.00$$

Thus

$$H = (V,\ E_H,\ F,\ P)$$

is a Behavior HyperGraph capturing both pairwise and multi-state co-occurrence patterns in workplace communication behavior.

Example 2.6 (Emotion Regulation Behavior HyperGraph)**.** Consider the set of emotion-regulation states from Gross's process model:

$$\begin{aligned}V = \{\,&\text{SituationSelection},\ \text{SituationModification},\ \text{AttentionalDeployment},\\ &\text{CognitiveChange},\ \text{ResponseModulation}\}.\end{aligned}$$

Suppose observational data yields the following transition frequencies $f : V \times V \to \mathbb{N}$:

$f(u,v)$	SitSel	SitMod	AttDep	CogCh	RespMod
SitSel	˘	10	0	0	0
SitMod	0	˘	12	0	0
AttDep	0	0	˘	15	0
CogCh	0	7	0	˘	8
RespMod	0	0	5	0	˘

All other $f(u,v) = 0$. The transition-probabilities $p(u,v) = f(u,v)/\sum_w f(u,w)$ then define the Behavior Graph

$$G = \big(V,\, E_G,\, f,\, p\big).$$

Its induced Behavior HyperGraph

$$H = \big(V,\, E_H,\, F,\, P\big)$$

has hyperedges

$$E_H = \big\{\{u,v\} : (u,v) \in E_G\big\} \,\cup\, \big\{\{\mathrm{CogCh}, \mathrm{RespMod}, \mathrm{AttDep}\}\big\} \,\cup\, \big\{\{\mathrm{AttDep}, \mathrm{SitMod}, \mathrm{CogCh}, \mathrm{RespMod}\}\big\}.$$

Here

$$F(e) \;=\; \sum_{\substack{(u,v)\in E_G \\ u,v\in e}} f(u,v), \qquad P(e) \;=\; \frac{F(e)}{\max_{e'\in E_H} F(e')}.$$

A few values are:

$F\big(\{\mathrm{SitMod}, \mathrm{AttDep}\}\big) = 12$ $\quad$ $P\big(\{\mathrm{SitMod}, \mathrm{AttDep}\}\big) = \frac{12}{47} \approx 0.255$

$F\big(\{\mathrm{CogCh}, \mathrm{RespMod}, \mathrm{AttDep}\}\big) = 8 + 7 + 15 + 5 = 35$ $\quad$ $P\big(\{\mathrm{CogCh}, \mathrm{RespMod}, \mathrm{AttDep}\}\big) = \frac{35}{47} \approx 0.745$

$F\big(\{\mathrm{AttDep}, \mathrm{SitMod}, \mathrm{CogCh}, \mathrm{RespMod}\}\big) = 12 + 15 + 7 + 8 + 5 = 47$ $\quad$ $P\big(\{\mathrm{AttDep}, \mathrm{SitMod}, \mathrm{CogCh}, \mathrm{RespMod}\}\big) = 1.00$

Thus

$$H = (V,\, E_H,\, F,\, P)$$

is a Behavior HyperGraph capturing both pairwise transitions and multi-state co-occurrence patterns in emotion-regulation behavior.

Theorem 2.7. *Every Behavior Graph $G = (V, E_G, f, p)$ admits a natural Behavior HyperGraph $H = (V, E_H, F, P)$, and the 2–section (clique-expansion) of H recovers G.*

Proof: Define

$$E_H \;=\; \big\{\, e \subseteq V :\; |e| \geq 2 \text{ and } \exists (u,v) \in E_G,\; u, v \in e \big\}.$$

Each e groups together states that co-occur in at least one transition of G. By construction $E_H \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.

For each $e \in E_H$, let

$$F(e) = \sum_{\substack{(u,v)\in E_G \\ u,v\in e}} f(u,v), \qquad P(e) = \frac{F(e)}{\max_{e'\in E_H} F(e')}.$$

Form the 2–section (clique-expansion) graph $\Gamma(H) = (V, E')$ by

$$E' = \big\{\{u,v\} \subseteq V : \exists e \in E_H,\; \{u,v\} \subseteq e\big\}.$$

If $(u,v) \in E_G$, then $\{u,v\} \subseteq e$ for $e = \{u,v\} \in E_H$, so $\{u,v\} \in E'$. Conversely, any $\{u,v\} \in E'$ arises from some $e \in E_H$, and thus $(u,v) \in E_G$ or $(v,u) \in E_G$. Hence $\Gamma(H)$ coincides with the undirected support of G. □

Theorem 2.8. *Every Behavior HyperGraph $H = (V, E_H, F, P)$ is a hypergraph in the sense of Definition 1.5.*

Proof: By definition $E_H \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$. Therefore H satisfies the hypergraph axioms immediately. □

Definition 2.9 (Behavior n-SuperHyperGraph)**.** Let

$$G = (V, E_G, f, p)$$

be a Behavior Graph and let

$$H = (V, E_H, F, P)$$

be its induced Behavior HyperGraph. For any integer $n \geq 1$, the *Behavior n-SuperHyperGraph* is defined as the tuple

$$\mathrm{BHG}^{(n)} = \big(V_n,\ E_n,\ F^{(n)},\ P^{(n)}\big),$$

where

$$V_n = \mathcal{P}^n(V), \qquad E_n = \big\{\, \mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\} \;\big|\; e \in E_H \big\},$$

and the label functions are propagated by

$$F^{(n)}\big(\mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\}\big) = F(e), \quad P^{(n)}\big(\mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\}\big) = P(e).$$

Example 2.10 (Emotion Regulation 2-SuperHyperGraph)**.** Let the Behavior Graph $G = (V, E_G, f, p)$ and its induced Behavior HyperGraph

$$H = (V, E_H, F, P)$$

be as in Example 2.6, where

$$V = \{\, \mathrm{SitSel},\ \mathrm{SitMod},\ \mathrm{AttDep},\ \mathrm{CogCh},\ \mathrm{RespMod}\},$$

and

$$E_H = \big\{\{\mathrm{SitMod}, \mathrm{AttDep}\},\ \{\mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\},$$
$$\{\mathrm{SitMod}, \mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}\big\},$$

with aggregate frequencies

$$F(\{\mathrm{SitMod}, \mathrm{AttDep}\}) = 12,$$
$$F(\{\mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}) = 15 + 8 + 5 = 28,$$
$$F(\{\mathrm{SitMod}, \mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}) = 12 + 15 + 8 + 5 + 7 = 47,$$

and normalized weights $P(e) = F(e)/47$.

For $n = 2$, the Behavior 2-SuperHyperGraph is

$$\mathrm{BHG}^{(2)} = \big(V_2,\ E_2,\ F^{(2)},\ P^{(2)}\big),$$

where

$$V_2 = \mathcal{P}^2(V),$$
$$E_2 = \big\{\mathcal{P}(e) \setminus \{\emptyset\} : e \in E_H\big\},$$

and for each hyperedge $e \in E_H$,

$$F^{(2)}\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) = F(e), \quad P^{(2)}\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) = P(e).$$

Two representative level-2 superedges are:

- *Pair superedge at SituationModification–AttentionalDeployment:*
$$\mathcal{P}\big(\{\mathrm{SitMod}, \mathrm{AttDep}\}\big) \setminus \{\emptyset\} = \{\{\mathrm{SitMod}\}, \{\mathrm{AttDep}\}, \{\mathrm{SitMod}, \mathrm{AttDep}\}\},$$
  carrying $F^{(2)} = 12$ and $P^{(2)} = 12/47 \approx 0.255$.
- *Junction superedge at the triple cluster* $\{\mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}$*:*
$$\mathcal{P}\big(\{\mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}\big) \setminus \{\emptyset\}$$
$$= \{\{\mathrm{AttDep}\}, \{\mathrm{CogCh}\}, \{\mathrm{RespMod}\}, \{\mathrm{AttDep}, \mathrm{CogCh}\}, \{\mathrm{AttDep}, \mathrm{RespMod}\}, \{\mathrm{CogCh}, \mathrm{RespMod}\},$$
$$\{\mathrm{AttDep}, \mathrm{CogCh}, \mathrm{RespMod}\}\},$$
  carrying $F^{(2)} = 28$ and $P^{(2)} = 28/47 \approx 0.596$.

These level-2 superedges unpack each hyperedge into all its nonempty subcollections, enabling hierarchical analysis of emotion-regulation sequences at multiple scales.

Example 2.11 (Online Learning Behavior 2-SuperHyperGraph)**.** Consider the set of behavioral states in an online course:

$$V = \{\text{VideoLecture}, \ \text{Quiz}, \ \text{ForumDiscussion}, \ \text{AssignmentSubmission}, \ \text{PeerReview}\}.$$

From learning-management-system logs, we observe the following transition frequencies $f : V \times V \to \mathbb{N}$:

$f(u,v)$	VideoLecture	Quiz	ForumDiscussion	AssignmentSubmission	PeerReview
VideoLecture	˘	50	20	10	0
Quiz	40	˘	15	25	5
ForumDiscussion	10	5	˘	30	15
AssignmentSubmission	0	10	20	˘	25
PeerReview	0	5	10	30	˘

All other $f(u,v)$ are zero. The induced Behavior HyperGraph

$$H = (V, \, E_H, \, F, \, P)$$

has hyperedges

$$\begin{aligned} E_H = \{&\{u,v\} : (u,v) \in E_G\} \\ &\cup \{\{\text{VideoLecture}, \text{Quiz}, \text{ForumDiscussion}\}\} \cup \{\{\text{Quiz}, \text{ForumDiscussion}, \text{AssignmentSubmission}\}\} \\ &\cup \{\{\text{ForumDiscussion}, \text{AssignmentSubmission}, \text{PeerReview}\}\}, \end{aligned}$$

grouping states with two or more common transitions. The aggregate frequency and normalized weight are

$$F(e) = \sum_{\substack{(u,v) \in E_G \\ u,v \in e}} f(u,v), \quad P(e) = \frac{F(e)}{\max_{e' \in E_H} F(e')}.$$

For instance,

$$F(\{\text{VideoLecture}, \text{Quiz}\}) = 50 + 40 = 90, \quad P(\{\text{VideoLecture}, \text{Quiz}\}) = \tfrac{90}{185} \approx 0.486,$$

$$F(\{\text{VideoLecture}, \text{Quiz}, \text{ForumDiscussion}\}) = 50 + 40 + 20 + 15 + 5 + 10 = 140, \quad P = 140/185 \approx 0.757.$$

Hence the Behavior 2-SuperHyperGraph

$$\mathrm{BHG}^{(2)} = (V_2, \, E_2, \, F^{(2)}, \, P^{(2)}),$$

with

$$V_2 = \mathcal{P}^2(V),$$
$$E_2 = \{\mathcal{P}(e) \setminus \{\emptyset\} : e \in E_H\},$$

and $F^{(2)}(\mathcal{P}(e) \setminus \{\emptyset\}) = F(e)$, $P^{(2)}(\mathcal{P}(e) \setminus \{\emptyset\}) = P(e)$, unpacks each hyperedge into its nonempty subcollections. This hierarchical model enables multi-scale analysis of student activity patterns in online learning.

Theorem 2.12. *For any Behavior HyperGraph $H = (V, E_H, F, P)$ and integer $n \geq 1$:*

(1) $\mathrm{BHG}^{(n)}$ is an n-SuperHyperGraph.

(2) $\mathrm{BHG}^{(1)}$ recovers H exactly, i.e. it generalizes the Behavior HyperGraph.

Proof: **(1) n-SuperHyperGraph structure.** By construction

$$V_n = \mathcal{P}^n(V), \qquad E_n \subseteq \mathcal{P}\big(V_n\big) \setminus \{\emptyset\}.$$

Hence $\mathrm{BHG}^{(n)} = (V_n, E_n, F^{(n)}, P^{(n)})$ satisfies the definition of an n-SuperHyperGraph.

(2) Generalization of H. When $n = 1$, note $\mathcal{P}^0(e) = e$ for each $e \in E_H$. Therefore

$$V_1 = \mathcal{P}(V), \qquad E_1 = \big\{ e \mid e \in E_H \big\} = E_H.$$

Moreover, for each $e \in E_H$,

$$F^{(1)}(e) = F(e), \quad P^{(1)}(e) = P(e).$$

There is a natural bijection $\iota : V \to \{\{v\} : v \in V\} \subset V_1$, identifying each original state v with its singleton $\{v\}$. Under this identification, $\mathrm{BHG}^{(1)}$ is isomorphic to H. Thus $\mathrm{BHG}^{(1)}$ recovers and generalizes the Behavior HyperGraph. □

3| Semantic Graph and Their Extensions

A *Semantic Graph* models concepts as nodes and meaning-based associations as edges, capturing the structure of mental organization and processes such as memory retrieval (cf. [35, 36, 37, 38, 39]). Semantic graphs are utilized not only in psychology but also in the fields of social science and artificial intelligence (cf.[40, 41, 42]). In this section, we explore their extensions using HyperGraphs and SuperHyperGraphs.

Definition 3.1 (Semantic Graph)**.** A *Semantic Graph* is a directed, labeled, and weighted graph

$$G = (V, E, L, \ell, w),$$

where

- V is a finite set of *concepts* or *terms*,
- $E \subseteq V \times V$ is the set of directed *edges* representing semantic relations,
- L is a finite set of *relation labels* (e.g. `is_a`, `part_of`, `associated_with`),
- $\ell : E \to L$ is a labeling function that assigns to each edge its semantic relation type,
- $w : E \to [0, 1]$ is a weight function that assigns to each edge a nonnegative real value reflecting the strength or confidence of the relation.

Example 3.2 (Mental Health Symptom Semantic Graph)**.** Let

$$V = \{\,\mathrm{Stress},\ \mathrm{Anxiety},\ \mathrm{Depression},\ \mathrm{Coping},\ \mathrm{SocialSupport}\},$$

$$L = \{\,\texttt{triggers},\ \texttt{contributes\_to},\ \texttt{enhances},\ \texttt{mitigates}\,\},$$

and define the directed edge set

$$E = \bigl\{\,(\mathrm{Stress}, \mathrm{Anxiety}),\ (\mathrm{Anxiety}, \mathrm{Depression}),\ (\mathrm{SocialSupport}, \mathrm{Coping}),\ (\mathrm{Coping}, \mathrm{Depression})\bigr\}.$$

Assign relation labels by

$$\ell : \begin{cases} (\mathrm{Stress}, \mathrm{Anxiety}) \mapsto \texttt{triggers}, \\ (\mathrm{Anxiety}, \mathrm{Depression}) \mapsto \texttt{contributes\_to}, \\ (\mathrm{SocialSupport}, \mathrm{Coping}) \mapsto \texttt{enhances}, \\ (\mathrm{Coping}, \mathrm{Depression}) \mapsto \texttt{mitigates}, \end{cases}$$

and weights

$$w : \begin{cases} (\mathrm{Stress}, \mathrm{Anxiety}) \mapsto 0.85, \\ (\mathrm{Anxiety}, \mathrm{Depression}) \mapsto 0.75, \\ (\mathrm{SocialSupport}, \mathrm{Coping}) \mapsto 0.80, \\ (\mathrm{Coping}, \mathrm{Depression}) \mapsto 0.65. \end{cases}$$

Then

$$G = \bigl(V, E, L, \ell, w\bigr)$$

is a Semantic Graph modeling key associations among mental health constructs in social-science research.

Example 3.3 (Intergroup Relations Semantic Graph)**.** Let

$$V = \{\,\mathrm{SocialIdentity},\ \mathrm{SelfEsteem},\ \mathrm{GroupCohesion},\ \mathrm{IntergroupConflict}\},$$

$$L = \{\,\texttt{influences},\ \texttt{mediates},\ \texttt{moderates}\,\},$$

and

$$E = \bigl\{\,(\mathrm{SocialIdentity}, \mathrm{SelfEsteem}),\ (\mathrm{SelfEsteem}, \mathrm{GroupCohesion}),\ (\mathrm{GroupCohesion}, \mathrm{IntergroupConflict}),$$

$$(\text{SocialIdentity}, \text{GroupCohesion}),\ (\text{SelfEsteem}, \text{IntergroupConflict})\big\}.$$

Define the labeling function $\ell : E \to L$ by

$$\ell : \begin{cases} (\text{SocialIdentity}, \text{SelfEsteem}) \mapsto \texttt{influences}, \\ (\text{SelfEsteem}, \text{GroupCohesion}) \mapsto \texttt{mediates}, \\ (\text{GroupCohesion}, \text{IntergroupConflict}) \mapsto \texttt{influences}, \\ (\text{SocialIdentity}, \text{GroupCohesion}) \mapsto \texttt{moderates}, \\ (\text{SelfEsteem}, \text{IntergroupConflict}) \mapsto \texttt{mediates}, \end{cases}$$

and the weight function $w : E \to [0, 1]$ by

$$w : \begin{cases} (\text{SocialIdentity}, \text{SelfEsteem}) \mapsto 0.85, \\ (\text{SelfEsteem}, \text{GroupCohesion}) \mapsto 0.75, \\ (\text{GroupCohesion}, \text{IntergroupConflict}) \mapsto 0.65, \\ (\text{SocialIdentity}, \text{GroupCohesion}) \mapsto 0.70, \\ (\text{SelfEsteem}, \text{IntergroupConflict}) \mapsto 0.60. \end{cases}$$

Then

$$G = \big(V, E, L, \ell, w\big)$$

is a Semantic Graph modeling core associations among constructs in intergroup relations research.

A Semantic HyperGraph represents concepts as vertices, linking multiple related concepts via hyperedges to capture contextual meaning. A Semantic SuperHyperGraph extends semantic hypergraphs, forming higher-order supervertices and superedges through iterated powersets for hierarchical semantic representation. In this section, we explore their extensions using HyperGraphs and SuperHyperGraphs.

Definition 3.4 (Semantic HyperGraph)**.** Let

$$G = (V, E, L, \ell, w)$$

be a Semantic Graph. Its *Semantic HyperGraph* is the tuple

$$H = \big(V,\ E_H,\ L_H,\ \ell_H,\ w_H\big),$$

where

$$E_H = \big\{\{u, v\} : (u, v) \in E\big\}\ \cup\ \big\{\{c\} \cup N_c^{\text{out}} : |N_c^{\text{out}}| \geq 2\big\}$$
$$\cup\ \big\{\{c\} \cup N_c^{\text{in}} : |N_c^{\text{in}}| \geq 2\big\},$$

with

$$N_c^{\text{out}} = \{\, v : (c, v) \in E\}, \qquad N_c^{\text{in}} = \{\, u : (u, c) \in E\}.$$

The hyperedge label set is

$$L_H = \mathcal{P}(L) \setminus \{\emptyset\},$$

and the labeling function $\ell_H : E_H \to L_H$ is

$$\ell_H(e) = \{\ \ell(u, v)\ :\ (u, v) \in E,\ \{u, v\} \subseteq e\}.$$

The weight function $w_H : E_H \to [0, 1]$ is

$$w_H(e) = \max_{\substack{(u,v) \in E \\ \{u,v\} \subseteq e}} w(u, v).$$

Example 3.5 (Socio-Political Semantic HyperGraph)**.** Let the underlying Semantic Graph $G = (V, E, L, \ell, w)$ be defined by

$$V = \{\,\text{SocialCapital},\ \text{Trust},\ \text{MediaInfluence},$$
$$\text{PoliticalParticipation},\ \text{SocialCohesion},\ \text{EconomicInequality}\},$$

relation labels

$$L = \{\ \texttt{influences},\ \texttt{mediates},\ \texttt{increases},\ \texttt{reduces}\ \},$$

and directed edges

$$E = \big\{\ (\text{SocialCapital}, \text{PoliticalParticipation}),$$

$$\begin{aligned}&(\text{Trust}, \text{PoliticalParticipation}),\ (\text{MediaInfluence}, \text{PoliticalParticipation}),\\&(\text{SocialCapital}, \text{SocialCohesion}),\ (\text{EconomicInequality}, \text{SocialCohesion})\big\}.\end{aligned}$$

Assign labels by

$$\ell : \begin{cases}(\text{SocialCapital}, \text{PoliticalParticipation}) \mapsto \texttt{influences},\\(\text{Trust}, \text{PoliticalParticipation}) \mapsto \texttt{mediates},\\(\text{MediaInfluence}, \text{PoliticalParticipation}) \mapsto \texttt{influences},\\(\text{SocialCapital}, \text{SocialCohesion}) \mapsto \texttt{increases},\\(\text{EconomicInequality}, \text{SocialCohesion}) \mapsto \texttt{reduces},\end{cases}$$

and weights by

$$w : \begin{cases}(\text{SocialCapital}, \text{PoliticalParticipation}) \mapsto 0.75,\\(\text{Trust}, \text{PoliticalParticipation}) \mapsto 0.65,\\(\text{MediaInfluence}, \text{PoliticalParticipation}) \mapsto 0.80,\\(\text{SocialCapital}, \text{SocialCohesion}) \mapsto 0.70,\\(\text{EconomicInequality}, \text{SocialCohesion}) \mapsto 0.60.\end{cases}$$

Then the induced Semantic HyperGraph

$$H = \big(V,\, E_H,\, L_H,\, \ell_H,\, w_H\big)$$

has hyperedges

$$\begin{aligned}E_H = {} & \{\{u, v\} : (u, v) \in E\}\\&\cup\ \{\{\text{SocialCapital}, \text{PoliticalParticipation}, \text{SocialCohesion}\}\}\\&\cup\ \{\{\text{SocialCapital}, \text{Trust}, \text{MediaInfluence}, \text{PoliticalParticipation}\}\}\\&\cup\ \{\{\text{SocialCapital}, \text{EconomicInequality}, \text{SocialCohesion}\}\}.\end{aligned}$$

The hyperedge label set is

$$L_H = \mathcal{P}(L) \setminus \{\emptyset\},$$

and for each $e \in E_H$,

$$\begin{aligned}\ell_H(e) &= \{\ell(u, v) \mid (u, v) \in E,\ \{u, v\} \subseteq e\},\\w_H(e) &= \max\{w(u, v) \mid (u, v) \in E,\ \{u, v\} \subseteq e\}.\end{aligned}$$

Hence, for example,

$$\begin{aligned}\ell_H(\{\text{SocialCapital}, \text{PoliticalParticipation}, \text{SocialCohesion}\}) &= \{\texttt{influences}, \texttt{increases}\},\\w_H(\{\text{SocialCapital}, \text{PoliticalParticipation}, \text{SocialCohesion}\}) &= 0.75,\end{aligned}$$

and

$$\begin{aligned}\ell_H(\{\text{SocialCapital}, \text{Trust}, \text{MediaInfluence}, \text{PoliticalParticipation}\})&\\= \{\texttt{influences}, \texttt{mediates}\},&\\w_H(\{\text{SocialCapital}, \text{Trust}, \text{MediaInfluence}, \text{PoliticalParticipation}\}) = 0.80.&\end{aligned}$$

This Semantic HyperGraph captures both pairwise and multi-concept associations in socio-political cognition.

Example 3.6 (Mental Health Symptom Semantic HyperGraph)**.** Let the underlying Semantic Graph $G = (V, E, L, \ell, w)$ be as in Example 3.2, with

$$V = \{\,\text{Stress},\ \text{Anxiety},\ \text{Depression},\ \text{Coping},\ \text{SocialSupport}\},$$

relation labels

$$L = \{\ \texttt{triggers},\ \texttt{contributes\_to},\ \texttt{enhances},\ \texttt{mitigates}\ \},$$

and directed edges

$$E = \big\{\ (\text{Stress}, \text{Anxiety}),\ (\text{Anxiety}, \text{Depression}),\ (\text{SocialSupport}, \text{Coping}),\ (\text{Coping}, \text{Depression})\big\}.$$

Then the *Semantic HyperGraph*

$$H = \big(V,\ E_H,\ L_H,\ \ell_H,\ w_H\big)$$

is defined by

$$E_H = \big\{\{u, v\} : (u, v) \in E\big\}\ \cup\ \big\{\{\,\text{Depression}, \text{Anxiety}, \text{Coping}\}\big\},$$

since Depression has two distinct semantic predecessors Anxiety and Coping. The hyperedge label set is

$$L_H = \mathcal{P}(L) \setminus \{\emptyset\},$$

and for each $e \in E_H$ we set

$$\ell_H(e) = \{\ \ell(u,v)\ :\ (u,v) \in E,\ \{u,v\} \subseteq e\},$$

$$w_H(e) = \max_{\substack{(u,v)\in E \\ \{u,v\}\subseteq e}} w(u,v).$$

Hence:

Hyperedge	**Labels**	**Weight**
$\{$Stress, Anxiety$\}$	$\{\texttt{triggers}\}$	0.85
$\{$Anxiety, Depression$\}$	$\{\texttt{contributes\_to}\}$	0.75
$\{$SocialSupport, Coping$\}$	$\{\texttt{enhances}\}$	0.80
$\{$Coping, Depression$\}$	$\{\texttt{mitigates}\}$	0.65
$\{$Depression, Anxiety, Coping$\}$	$\{\texttt{contributes\_to}, \texttt{mitigates}\}$	0.75

This Semantic HyperGraph H groups co-occurring concepts and identifies the junction hyperedge at Depression, capturing multi-concept associations in mental-health cognition.

Theorem 3.7. *The Semantic HyperGraph $H = (V, E_H, L_H, \ell_H, w_H)$ is a hypergraph, and its 2-section (clique-expansion) $\Gamma(H)$ recovers the undirected support of the original Semantic Graph G.*

Proof: First, by construction $E_H \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so H satisfies the definition of a hypergraph.

Next, form the 2-section graph $\Gamma(H) = (V, E')$ where

$$E' = \big\{\{u,v\} \subseteq V : \exists\, e \in E_H,\ \{u,v\} \subseteq e\big\}.$$

If $(u,v) \in E$, then $\{u,v\} \in E_H$ by definition, so $\{u,v\} \in E'$. Conversely, if $\{u,v\} \in E'$, there exists $e \in E_H$ with $\{u,v\} \subseteq e$. If $e = \{u,v\}$, then $(u,v) \in E$ or $(v,u) \in E$. Otherwise $e = \{c\} \cup N$ for some c, and $\{u,v\} \subseteq e$ implies one of u, v equals c and the other lies in N, so again $(u,v) \in E$ or $(v,u) \in E$. Thus E' coincides with the set of unordered pairs arising from edges of G, i.e. the undirected support of G. □

Definition 3.8 (Semantic n-SuperHyperGraph)**.** Let

$$G = (V, E, L, \ell, w)$$

be a Semantic Graph, and let

$$H = (V, E_H, L_H, \ell_H, w_H)$$

be its induced Semantic HyperGraph. For any integer $n \geq 1$, the *Semantic n-SuperHyperGraph* is the tuple

$$\mathrm{SNHG}^{(n)} = \big(V_n,\ E_n,\ L_n,\ \ell_n,\ w_n\big),$$

where

$$V_n = \mathcal{P}^n(V),$$

$$E_n = \big\{\ \mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\}\ \big|\ e \in E_H\big\},$$

$$L_n = \mathcal{P}(L_H) \setminus \{\emptyset\},$$

$$\ell_n\big(\mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\}\big) = \ell_H(e),$$

$$w_n\big(\mathcal{P}^{\,n-1}(e) \setminus \{\emptyset\}\big) = w_H(e).$$

Example 3.9 (Mental Health Symptom Semantic 2-SuperHyperGraph)**.** Let the underlying Semantic Graph $G = (V, E, L, \ell, w)$ be as in Example 3.2, where

$$V = \{\text{Stress},\ \text{Anxiety},\ \text{Depression},\ \text{Coping},\ \text{SocialSupport}\},$$

and the induced Semantic HyperGraph

$$H = (V,\ E_H,\ L_H,\ \ell_H,\ w_H)$$

has hyperedges

$$E_H = \big\{ \{\text{Stress}, \text{Anxiety}\},\ \{\text{Anxiety}, \text{Depression}\},\ \{\text{SocialSupport}, \text{Coping}\},$$

$$\{\text{Coping}, \text{Depression}\},\ \{\text{Anxiety}, \text{Depression}, \text{Coping}\}\big\}.$$

Then the *Semantic 2-SuperHyperGraph* $\mathrm{SNHG}^{(2)} = (V_2, E_2, L_2, \ell_2, w_2)$ is defined by

$$\begin{aligned} V_2 &= \mathcal{P}^2(V), \\ E_2 &= \big\{ \mathcal{P}(e) \setminus \{\emptyset\} \bigm| e \in E_H \big\}, \\ L_2 &= \mathcal{P}(L_H) \setminus \{\emptyset\}, \\ \ell_2\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) &= \ell_H(e), \\ w_2\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) &= w_H(e). \end{aligned}$$

Two representative level-2 superedges are:

- **Trigger superedge:**

  $$\mathcal{P}\big(\{\text{Stress}, \text{Anxiety}\}\big) \setminus \{\emptyset\}$$

  $$= \big\{ \{\text{Stress}\},\ \{\text{Anxiety}\},\ \{\text{Stress}, \text{Anxiety}\}\big\},$$

  carrying labels $\ell_2 = \{\texttt{triggers}\}$ and weight $w_2 = 0.85$.

- **Junction superedge at Depression:**

  $$\mathcal{P}\big(\{\text{Anxiety}, \text{Depression}, \text{Coping}\}\big) \setminus \{\emptyset\}$$

  $$= \big\{\{\text{Anxiety}\},\ \{\text{Depression}\},\ \{\text{Coping}\},\ \{\text{Anxiety}, \text{Depression}\},\ \ldots,$$

  $$\{\text{Anxiety}, \text{Depression}, \text{Coping}\}\big\},$$

  with labels $\ell_2 = \{\texttt{contributes\_to}, \texttt{mitigates}\}$ and weight $w_2 = 0.75$.

These level-2 superedges unpack each hyperedge into all its nonempty subcollections, enabling hierarchical analysis of multi-concept associations in mental-health cognition.

Example 3.10 (Socioeconomic Semantic 2-SuperHyperGraph)**.** Let the base set of key socioeconomic concepts be

$$V = \{\text{Democracy},\ \text{EconomicGrowth},\ \text{Education},\ \text{SocialMobility},\ \text{IncomeInequality}\}.$$

Define the relation-label set

$$L = \{\ \texttt{promotes},\ \texttt{mediates},\ \texttt{reduces},\ \texttt{increases}\ \},$$

and the directed edge set

$$E = \big\{ (\text{Democracy}, \text{EconomicGrowth}),\ (\text{Education}, \text{SocialMobility}),$$

$$(\text{SocialMobility}, \text{IncomeInequality}),\ (\text{EconomicGrowth}, \text{IncomeInequality})\big\}.$$

Assign labels by

$$\ell : \begin{cases} (\text{Democracy}, \text{EconomicGrowth}) \mapsto \texttt{mediates}, \\ (\text{Education}, \text{SocialMobility}) \mapsto \texttt{promotes}, \\ (\text{SocialMobility}, \text{IncomeInequality}) \mapsto \texttt{reduces}, \\ (\text{EconomicGrowth}, \text{IncomeInequality}) \mapsto \texttt{increases}, \end{cases}$$

and weights by

$$w: \begin{cases} (\text{Democracy}, \text{EconomicGrowth}) \mapsto 0.70, \\ (\text{Education}, \text{SocialMobility}) \mapsto 0.80, \\ (\text{SocialMobility}, \text{IncomeInequality}) \mapsto 0.60, \\ (\text{EconomicGrowth}, \text{IncomeInequality}) \mapsto 0.75. \end{cases}$$

Then $G = (V, E, L, \ell, w)$ is a Semantic Graph modeling these constructs.

Its induced Semantic HyperGraph

$$H = (V, E_H, L_H, \ell_H, w_H)$$

has hyperedges

$$E_H = \big\{\{u, v\} : (u, v) \in E\big\} \cup$$

$$\big\{\{\text{EconomicGrowth}, \text{SocialMobility}, \text{IncomeInequality}\}\big\},$$

since IncomeInequality has two semantic predecessors. Here

$$L_H = \mathcal{P}(L) \setminus \{\emptyset\},$$

$$\ell_H(e) = \{\ell(u, v) : (u, v) \in E, \{u, v\} \subseteq e\},$$

$$w_H(e) = \max_{\substack{(u,v)\in E \\ \{u,v\}\subseteq e}} w(u, v).$$

For $n = 2$, the *Semantic 2-SuperHyperGraph* is

$$\mathrm{SNHG}^{(2)} = \big(V_2, E_2, L_2, \ell_2, w_2\big),$$

where

$$V_2 = \mathcal{P}^2(V),$$

$$E_2 = \big\{\mathcal{P}(e) \setminus \{\emptyset\} \mid e \in E_H\big\},$$

$$L_2 = \mathcal{P}(L_H) \setminus \{\emptyset\},$$

$$\ell_2\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) = \ell_H(e),$$

$$w_2\big(\mathcal{P}(e) \setminus \{\emptyset\}\big) = w_H(e).$$

Two illustrative level-2 superedges are:

$$\mathcal{P}\big(\{\text{Education}, \text{SocialMobility}\}\big) \setminus \{\emptyset\}$$

$$= \big\{\{\text{Education}\},\ \{\text{SocialMobility}\},\ \{\text{Education}, \text{SocialMobility}\}\big\},$$

with labels $\{\texttt{promotes}\}$ and weight 0.80; and

$$\mathcal{P}\big(\{\text{EconomicGrowth}, \text{SocialMobility}, \text{IncomeInequality}\}\big) \setminus \{\emptyset\}$$

$$= \big\{\ \{\text{EconomicGrowth}\},\ \{\text{SocialMobility}\},\ \{\text{IncomeInequality}\},\ \dots\big\},$$

with labels $\{\texttt{increases}, \texttt{reduces}\}$ and weight 0.75.

These superedges unpack each hyperedge into all nonempty subcollections, enabling hierarchical analysis of complex concept associations in social-science contexts.

Theorem 3.11. *For any Semantic HyperGraph $H = (V, E_H, L_H, \ell_H, w_H)$ and integer $n \geq 1$:*

(1) $\mathrm{SNHG}^{(n)}$ satisfies the definition of an n-SuperHyperGraph.

(2) $\mathrm{SNHG}^{(1)}$ recovers H up to hypergraph isomorphism.

Proof: **(1) n-SuperHyperGraph structure.** By construction,

$$V_n = \mathcal{P}^n(V), \quad E_n \subseteq \mathcal{P}(V_n) \setminus \{\emptyset\}.$$

Hence $\mathrm{SNHG}^{(n)} = (V_n, E_n, L_n, \ell_n, w_n)$ meets the axioms of an n-SuperHyperGraph.

(2) Generalization of H. When $n = 1$, $\mathcal{P}^0(e) = e$ for each $e \in E_H$. Therefore

$$V_1 = \mathcal{P}(V), \qquad E_1 = \{\, e : e \in E_H \} = E_H.$$

Moreover, $\ell_1(e) = \ell_H(e)$ and $w_1(e) = w_H(e)$ for all e. The natural bijection $\iota : V \to \{\{v\} : v \in V\} \subset V_1$ identifies each original vertex v with its singleton $\{v\}$. Under ι, $\mathrm{SNHG}^{(1)}$ is isomorphic to H. Thus $\mathrm{SNHG}^{(1)}$ exactly recovers and generalizes the Semantic HyperGraph. □

4| Conclusion and Future Work

In this paper, we have extended Behavior Graphs and Semantic Graphs using HyperGraphs and SuperHyperGraphs to produce hierarchical, multi-scale representations of cognitive and behavioral phenomena.

As future work, we plan to investigate further extensions by incorporating fuzzy and neutrosophic frameworks, including Fuzzy Sets [43, 44, 45], HyperFuzzy Sets [46, 47], Picture Fuzzy Sets [48, 49], Hesitant Fuzzy Sets [50, 51], Neutrosophic Sets [52, 53], and Plithogenic Sets [54, 55].

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Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Ethical Approval

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

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